ESD-TDR-64-452

ELECTRONIC SYSTEMS DIVISION
PRECISION ORBIT DETERMINATION PROGRAM: THIRD QUARTERLY REPORT

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-452

15 April 1964

496L Systems Program Office
Electronic Systems Division
Air Force Systems Command

United States Air Force
L.G. Hanscom Field, Bedford, Massachusetts

Prepared Under Contract AF 19(628)-594

STL No. 8497-6070-RU000

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THIRD QUARTERLY REPORT

REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.

William M. Robertson

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1/LT USAF
Assistant Contract Monitor

FOREWORD

This is the third and last quarterly report prepared under contract AF 19(628)-594 for the Electronic Systems Division, United States Air Force System Command, L. G. Hanscom Field, Bedford, Massachusetts. Previous quarterly reports described the technical performance agreed to and planned under the contract. Full description of the computer program ESPOD developed under this contract appears in other contract documentation.

This report was prepared by TRW Space Technology Laboratories, One Space Park, Redondo Beach, California.

INTRODUCTION

This Third Quarterly Report under Contract AF 19(628)-594 covers certain technical and mathematical considerations which are germane to the accuracy of ESPOD. Particular emphasis is given to the numerical integration process, the calculation of the vernal equinox, and the interpretation of the earth potential model.

CONTENTS

1. NUMERICAL EXPERIMENTS WITH ESPOD INTEGRATION SUBROUTINE

This report presents the results of some numerical experiments conducted with the ESPOD integration subroutine.

The experiment consisted of integrating the equations of motion

$$\ddot{y} = \mu \frac{y}{|y|^3}, \qquad y = (y_1, y_2) \tag{1}$$

for long periods of time and then comparing the results to reference orbits. The reference orbits were calculated by means of analytic formulas for the two-body problem. In the case of circular orbits this was merely the appropriate use of a completely double precision sine-cosine subroutine. For other orbits, Kepler's equation was solved at each compared time point.

A comparison between the ESPOD integration subroutine and another subroutine which is currently in use in other tracking programs has been included. The ESPOD integration subroutine is a 10^{th} order Cowell scheme using single precision arithmetic. This routine was written especially for the Philco 2000 computer and the lack of multiple precision is partially made up for by the larger word length (10 decimal digits) of this machine. A version of this same subroutine was written for the 7094 using partial double precision. The subroutine called DE6F, upon which the Philco 2000 routine is modeled, is an 8^{th} order Cowell scheme using partial double precision arithmetic. Besides the order of the highest retained differences, the other principal difference between these subroutines is that DE6F restarts when it changes step-size while the Cowell method retabulates the differences.

1.1 CASE 1

The first orbit was a circular orbit with r = 1.029034544 er, which is 2.15333073×10^7 feet or about 115 miles above the earth. Such an orbit will have a period P of 88.19557292 minutes. Equation (1) was integrated for about 16,000 minutes or 181 orbits at a fixed step-size of 2 minutes. The errors in the individual rectangular components were sampled during the first 15 orbits to establish their pattern of growth. This showed, as theory predicts, that the errors exhibit an oscillation whose amplitude grows nearly linearly with time. The errors were again sampled during the last three orbits. The maximum error in either of the in-plane components was less than 478 ft for these final orbits. The error in the in-plane angle

$$\theta = \tan^{-1} \frac{y_2}{y_1}$$

was sampled during the entire run. The error in θ at the end of the run was about 1×10^{-5} radians or about 6×10^{-4} degrees. The same run was made with a step-size of 4 minutes and the accumulated error in θ was found not to exceed 3.6×10^{-3} radians or about 2.1×10^{-1} degrees.

1.2 CASE 2

The second orbit for which errors have been computed is an ellipse of eccentricity e = 0.73130568 aligned with its major axis along the y_1 coordinate line. Its distance from the origin at perigee was 1.029865500 er and its period was P = 634.00369063 minutes. In this case, Equation (1) was integrated until 15 orbits were completed or for about 8900 minutes. Table 1 gives the error δ observed in the rectangular components y_1 and y_2 for various subroutines during the 14th orbit. The error is quoted in feet. It was inconvenient to obtain output at exactly the same time points on each run because the runs were made in the variable step mode. This accounts for the lack of entries in some places in the table. In the case of the ESPOD subroutines, the step-size varied between one-half minute near perigee to 8 minutes in the vicinity of apogee. In the case of DE6F the variation was one-half to 4 minutes. Table 2 gives a sample of the errors δ in the rectangular components (y_1 and y_2) and true anomaly f during the 15th orbit.

1.3 CASE 3

The third orbit for which errors have been computed is an ellipse of moderate eccentricity, e = approximately 0.015. The major axis was aligned with the y_1 coordinate axis. Perigee distance is 6774 km. Period is 90.7 minutes. The orbit was integrated for 300 revolutions, that is, for approximately 18.8 days or 28,000 minutes. Reference points were calculated for comparison from an STL analytic program. On the last orbit the maximum error in either of the two in-plane components was less than 5.7 km.

Table 1. Y-Component Errors Observed During 14th Orbit

	ESPOD on the PHILCO 2000		ESPOD on 7094		DE6F (parent routine)	
T (min)	$\lvert\delta Y_1\rvert$ ft	$\lvert\delta Y_2\rvert$ ft	$\lvert\delta Y_1\rvert$ ft	$\lvert\delta Y_2\rvert$ ft	$\lvert\delta Y_1\rvert$ ft	$\lvert\delta Y_2\rvert$ ft
8244.5	176	1481			16, 405	121, 608
8250	527	1298	349	727		
8254.5	696	1097			60, 312	90, 491
8256	815	705	552	400		
8280	766	387	544	195		
8285	738	322			66, 153	24, 630
8295	678	224	504	73	82, 594	15, 571
8309	603	128			55, 656	6, 834
8331	504	38			47, 401	1, 883
8355	416	23	349	123		
8379	341	61				
8415	249	96	241	190	25, 079	15, 644
8475	126	136	146	213		
8595	98	163	4	202		
8815	760	144			66, 851	12, 317
8850	938	655	381	287	82, 402	58, 864

Table 2. Y Component and Phase Angle Error on the 15th Orbit

T (min)	$\lvert\delta Y_1\rvert$ ft	$\lvert\delta \dot{Y}_2\rvert$ ft	δf (seconds of arc)	True Anomaly (deg)
8865	420.8	747.1	6.318	308
8876	-18.9	971.3	9.305	359.72
8885	-424.1	821.5	7.473	44
9000	-353.4	-153.1	0.645	149
9162	-12.3	-247.4	0.351	179.86
9432	385.0	9.0	0.729	225

2. PRECESSION AND NUTATION OF THE EARTH FOR USE IN ESPOD

2.1 TERMINOLOGY

2.1.1 Luni-solar Precession

Long-term effects of Sun and Moon acting on the Earth's equatorial bulge which produce a conical rotation of the polar axis once every 26,000 years, or 50"/year.

2.1.2 Planetary Precession

Perturbation of the ecliptic plane by the planets causing a change in orientation of the ecliptic about a line at longitude 174 degrees. The effect on the vernal equinox is about $0\overset{''}{.}12$/year and on the obliquity of the ecliptic is about $-0\overset{''}{.}47$/year.

2.1.3 General Precession

The sum of the effects of luni-solar and planetary precession on equator and equinox.

2.1.4 Nutation

A relatively short period wobble of the Earth's pole superimposed on the precessional motion which results from luni-solar effects on the equatorial bulge. The major short period effect is of amplitude $0\overset{''}{.}09$ and period 13.6 days.

2.1.5 Mean Equator and Equinox

The plane perpendicular to the pole of the Earth including luni-solar precession is defined as the mean equator at a given date. The intersection of this plane with the ecliptic plane including planetary precession of the same date defines the related mean equinox. The system as a whole includes general precession but not nutation.

2.1.6 True Equator and Equinox

The plane perpendicular to the pole of the Earth including luni-solar precession and nutation is the true* equator at a given date. The

*Note that the true equator by definition corresponds to the actual equator of the Earth, i.e., that plane passing through the center of mass, perpendicular to the axis of rotation.

intersection of this plane with the ecliptic plane including planetary precession defines the true equinox. The system as a whole includes general precession and nutation.

2.1.7 Of Date Versus of Date of Epoch

The above defined true and mean systems may be either rotating or inertial. If the system is allowed to rotate with time to include changes in precession (and nutation for true system) it is termed "of date." If the system is held fixed at conditions for some arbitrary epoch (e.g., 1950.0, 0^h of date of launch, etc.) it is termed "of date of epoch," or "of epoch."

2.2 EQUATIONS AND ROTATION MATRICES

The fundamental time units and notations used herein are as follows:

Notation	Unit	Description
t	Ephemeris day	Julian ephemeris date
t_o	Ephemeris day	Julian ephemeris date of epoch
$t_{1950.0}$	Ephemeris day	Julian ephemeris date of 1950.0 = 2433281.923
T	Tropical century	$(t - t_o)/36524.220$
T_o	Tropical century	$(t_o - t_{1950.0})/36524.220$
d	Ephemeris day	t - 2415020.0 (days since 1900)
D	10,000 ephemeris days	(t - 2415020.0)/10,000
J	Tropical year	(t - 2415020.0)/365.24 (years since 1900)

The rotation from mean equator and equinox of epoch t_o to mean equator and equinox of date t is as follows:

$$\underline{r} = (I + P)\,\underline{r}_o$$

$$\underline{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\underset{\sim}{r}_o = \begin{pmatrix} x_o \\ y_o \\ z_o \end{pmatrix}$$

$$P = \begin{pmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{pmatrix}$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where for unit 10^{-8}:

$$P_{11} = -(29,696 + 26T_o)\,T^2 - 13T^3$$

$$P_{12} = -P_{21} = -(2,234,941 + 1355T_o)\,T - 676T^2 + 221T^3$$

$$P_{13} = -P_{31} = -(971,690 - 414T_o)\,T + 207T^2 + 96T^3$$

$$P_{22} = -(24,975 + 30T_o)\,T^2 - 15T^3$$

$$P_{23} = P_{32} = -(10,858 + 2T_o)\,T^2$$

$$P_{33} = -(4721 - 4T_o)\,T^2$$

The rotation from mean equator and equinox of date to true equator and equinox of date is as follows

$$\underline{r} = (I + N)\,\underset{\sim}{r}_o$$

$$N = \begin{pmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{pmatrix}$$

where

$$N_{11} = 0$$

$$N_{12} = -N_{21} = -\Delta\psi \cos \epsilon$$

$$N_{13} = -N_{31} = -\Delta\psi \sin \epsilon$$

$$N_{22} = 0$$

$$N_{23} = -N_{32} = -\Delta\epsilon$$

$$N_{33} = 0$$

$\Delta\psi$ = nutation in longitude of date

ϵ = obliquity of ecliptic of date

$\Delta\epsilon$ = nutation in obliquity of date

The nutations in longitude and obliquity are computed from sums of sine and cosine terms of the following form

$$\Delta\psi = \sum_i S_i \sin (a_i \ell + b_i \ell' + c_i F + d_i D + e_i \Omega)$$

$$\Delta\epsilon = \sum_i C_i \cos (a_i \ell + b_i \ell' + c_i F + d_i D + e_i \Omega)$$

where the arguments (ℓ, ℓ', F, D, Ω) are functions of time related to the motions of Moon and Sun

$$\ell = 296^\circ\!.104608 + 13^\circ\!.0649924465d + 0.0006890D^2 + 0^\circ\!.000000295D^3$$

$$\ell' = 358^\circ\!.475833 + 0.9856002669d + 0.0000112D^2 - 0.000000068D^3$$

$$F = 11^\circ\!.250889 + 13^\circ\!.2293504490d - 0.0002407D^2 - 0.000000007D^3$$

$$D = 350^\circ\!.737486 + 12^\circ\!.190749194d - 0^\circ\!.0001076D^2 + 0.000000039D^3$$

$$\Omega = 259^\circ\!.183275 - 0.0529539222d + 0^\circ\!.0001557D^2 + 0.000000046D^3$$

Note the ambiguity in notation where D is both a unit of time and an angular argument. The coefficients S_i, C_i, are grouped according to their magnitude and period in Table 3 with $a_i \cdots, e_i$, the integers multiplying the above arguments. Terms of magnitude $\leq 0''.006$ have been omitted, but may be found in Reference 2.5.1. Accuracies have been rounded off from $0''.0001$ (Reference 2.5.1) to $0''.01$.

For an accuracy reduced from $0''.01$ to $0''.1$, the table may be summarized by the following expressions

$$\Delta\psi = -17''.2 \sin \Omega + 0''.2 \sin 2\Omega - 1''.3 \sin (2F - 2D + 2\Omega)$$
$$+ 0''.1 \sin \ell' - 0''.2 \sin (2F + 2\Omega) + 0''.1 \sin \ell$$

$$\Delta\epsilon = 9''.2 \cos \Omega - 0''.1 \cos 2\Omega + 0''.6 \cos (2F - 2D + 2\Omega)$$
$$+ 0''.1 \cos (2F + 2\Omega)$$

The value for ϵ required to obtain $\sin \epsilon$ and $\cos \epsilon$ multipliers for $\Delta\psi$ in the N matrix is time variant. However, for a period from 1964 to 1976 the change in ϵ is only $6''.41$ and for the purposes of the computation of the $\Delta\psi \cos \epsilon$ and $\Delta\psi \sin \epsilon$ terms in the N matrix may be taken as

$$\epsilon = 23^\circ 26' 35'' \qquad \sin \epsilon = 0.39784 \qquad \cos \epsilon = 0.91745$$

with only a possible error of ±1 in the fifth decimal place for 1964 to 1976. For greater accuracy, consult Table 2.3 of Reference 2.5.1, which gives ecliptic elements through 1980.

Table 3. Coefficients of Nutation in Longitude and Obliquity

Unit (related argument)	Period (days)	a_i (ℓ)	b_i (ℓ')	c_i (F)	d_i (D)	e_i (Ω)	S_i (0."01) (sine, longitude)	C_i (0."01) (cosine, obliquity)
	6798					1	-1722.4	921
							$-0.017(T_o + T)$	
	3399					2	21	-9
	183			2	-2	2	-127	55
	365		1				13	
	122		1	2	-2	2	-5	2
	365		-1	2	-2	2	2	-1
	178			2	-2	1	1	-1
	13.7			2		2	-20	9
	27.5	1					7	
	13.6			2		1	-3	2
	9.1	1		2		2	-3	1
	31.8	1			-2		-1	
	27.1	-1		2		2	1	-1

The rotation from mean equator and equinox of epoch to true equator and equinox of date is the product of the nutation and precession rotations. For application in ESPOD (where $t - t_o < 30$ days) the second order term, NP, is negligible and

$$\underline{r} = (I + N)(I + P)\,\underline{r}_o$$

$$= (I + N + P)\,\underline{r}_o$$

Second order terms in general will not exceed an order of $2 \times 10^{-6}T$, where $T = (t - t_o)/36524$. Hence, they may become significant in the eight digit after six months and in the seventh after five years.

The limiting magnitudes of the nutation, 30-day precession, and 30-day nutation rotation matrices are presented below. The notation dN indicates the rotation from true of epoch to true of date (differential nutation), as opposed to N for mean of epoch to true of date (total nutation).

unit 10^{-6}

$$N < \begin{pmatrix} 0 & \pm 89 & \pm 39 \\ \mp 89 & 0 & (\pm)49 \\ \mp 39 & (\mp)49 & 0 \end{pmatrix}$$

$$\underset{(30\text{ day})}{P} < \begin{pmatrix} 0 & -18 & -8 \\ 18 & 0 & 0 \\ 8 & 0 & 0 \end{pmatrix}$$

$$\underset{(30\text{ day})}{dN} < \begin{pmatrix} 0 & \pm 6 & \pm 3 \\ \mp 6 & 0 & (\pm)3 \\ \mp 3 & (\mp)3 & 0 \end{pmatrix}$$

The $N_{32} = -N_{23}$ and $dN_{32} = -dN_{23}$ terms may be of opposite sign from the corresponding N_{12}, N_{13}, etc., terms; hence, the sign was enclosed in parentheses.

2.3 COORDINATE SYSTEMS USED IN ESPOD

2.3.1 Observations

Radar observations are by virtue of their connection to the actual position of the Earth in true equator equinox of date system. Baker-Nunn angular observations are available to ESPOD in varying coordinates depending on whether they are precision or field reduced and whether or not they are preprocessed by other programs in the SPADATS system before entering ESPOD. The applicable logic is given in the introduction to Section 7.1 of ESD-TDR-64-395 and the updating within ESPOD is described under the ADJUST subroutine in the same document.

2.3.2 Integration

The integration is carried on using the time equator and equinox of $0^h \cdot 0$ on the day of epoch. The right ascension of Greenwich at epoch is

$$\alpha_{go} = \frac{\pi}{43200}\left\{23925.836 + 1.84542J + 9.29 \times 10^{-6}J^2 + n + 2\pi\left(J - [J]\right)\right\}$$

$$J = \frac{t_o - t_{1900.0}}{365.25}$$

$$[J] = \text{greatest integer in } J$$

The expression n changes α_{go} to true equator and equinox of $0^h \cdot 0$ of the day of epoch. Both the rotation of the orbit to Earth-fixed coordinates for representation, and the computation of station longitude are dependent on α_{go} for determining the position of the Earth at a given time.

The computation for α_{go} includes the nutation correction which changes the station right ascensions and places the integration in true of $0^h \cdot 0$ of day of epoch (see subroutine TINIT). The precision and field-reduced angular observations are also corrected for nutation. The correction is made using $\Delta\psi$, $\Delta\epsilon$, at $0^h \cdot 0$ of day of epoch, to agree with the integration (see subroutine ADJUST). The adjusted values of the precision reduced angular observations will vary from true of epoch by a negligible $P(t - t_o)$, the precession from $0^h \cdot 0$ of day of epoch to the time of observation because of their original reference to mean of date.

2.3.3 Summary of Coordinate Systems

The coordinate systems after revision will be as follows:

Radar observations	true of date
Field-reduced observations Baker-Nunn	true of $0^h \cdot 0$ day of epoch
Precision reduced Baker-Nunn observations	true of $0^h \cdot 0$ day of epoch + $P(t - t_o)$
Integration	true of epoch
Station longitude	true of epoch
Station latitude	true of date = true

2.3.4 Bias Limits

In observed minus computed residuals

Type of Observation	Bias	Limiting 30-Day Magnitude
Radar	$P(t - t_o) + dN$	0.15 km
Field reduced angular	None	0
Precision angular	$P(t - t_o)$	0.11 km

2.4 APPROXIMATE NUTATION FORMULAS

Nutation in longitude, and obliquity at epoch, are obtained to within $0\rlap{.}''5$ from

$$\Delta\psi = -17\rlap{.}''2 \sin\Omega - 1\rlap{.}''3 \sin(2F - 2D + 2\Omega)$$

$$\Delta\epsilon = 9\rlap{.}''2 \cos\Omega + 0\rlap{.}''6 \cos(2F - 2D + 2\Omega)$$

$$\Omega = 259\rlap{.}^{\circ}18 - 19\rlap{.}^{\circ}3414\ J$$

$$2F - 2D + 2\Omega = -160\rlap{.}^{\circ}61 + 719\rlap{.}^{\circ}9957\ J$$

The correction n to α_{go} is then

$$n = \cos\epsilon\,\Delta\psi$$

and the corrections for nutation to the precision and field-reduced Baker-Nunn observations are

$$\Delta\alpha = (\cos\epsilon + \sin\epsilon \sin\alpha \tan\delta)\Delta\psi - \cos\alpha \tan\delta\Delta\epsilon$$

$$\Delta\delta = \sin\epsilon \cos\alpha\Delta\psi + \sin\alpha\Delta\epsilon$$

2.5 REFERENCES

2.5.1 Explanatory Supplement, American Ephemeris and Nautical Almanac, 1961.

2.5.2 ESD-TDR-64-395 ESPOD Mathematical and Subroutine Description.

3. GEOPOTENTIAL MODEL ACCURACIES

3.1 ACCURACIES OF SPADATS MODELS

Four coherent models of the earth potential field will be provided with ESPOD. These models are summarized in Table 4.

3.1.1 Model 1

Model 1 will give an accuracy within approximately 0.5 km of a period of 24 hours. The major error is caused by neglecting the second sectorial harmonic (with coefficient $J_{2,2}$ at the phase angle $\lambda_{2,2}$). The effect is sinusoidal with a period of 12 hours.

3.1.2 Model 2

Model 2 will give approximately the same accuracy as Model 1 for the same reason.

3.1.3 Model 4

Model 4 is estimated to provide an accuracy of approximately 0.2 km, with uncertainties in the sectorial and tesseral harmonics making further accuracy impossible.

3.2 REFERENCES

3.2.1 R.R. Newton, Results from Transit Satellite Tracking, reported in A.H. Cook, Space Science Reviews, 2, 1963.

This report indicated an amplitude 0.5 km for the $J_{2,2}$ perturbation.

3.2.2 R.R. Newton, Science, 15, 803-808.

A lower error limit of 0.10 km is estimated possible by eventual inclusion of 50 harmonic coefficients in a solution with ample and well distributed data.

3.2.3 Conclusions referenced in Paragraph 3.1.3 are an estimate by M.P. Francis from probable errors associated with Model 4 by Kozai, SAO Special Report No. 72, 9 August 1961.

Table 4. Earth Potential Model Options

n	m	Stored in Program		Model 1		Model 2		Model 3		Model 4	
		$J_{n,m} \times 10^{+6}$	λ	$J_{n,m} \times 10^6$	λ	$J_{n,m} \times 10^6$	λ	$J_{n,m} \times 10^6$	λ	$J_{n,m} \times 10^6$	λ
2		1082.30		1082.30		1082.48*		Use Model 2		Use Model 2	
3		-2.3		-2.3		-2.562*					
4		-1.8		-1.8		-1.84*					
5		-0.064		OMIT		-0.064					
6		0.390				0.390					
7		-0.470				-0.470					
8		-0.020				-0.020					
9		0.117				0.117					
2	2	2.32	-37.5			OMIT		2.32	-37.5	2.32	-37.5
3	1	3.95	22.							3.95	22.
3	2	0.41	31.							0.41	31.
3	3	1.91	51.3							1.91	51.3
4	1	2.64	163.5							2.64	163.5
4	2	1.67	54.							1.67	54.
4	3	0.46	-13.							0.46	-13.
4	4	0.56	50.3							0.56	50.3

3.3 TESTS FOR POTENTIAL HARMONICS

Tests have been run to determine the effect of omitting higher order harmonics from the earth's gravitational field. Although the tests were not run for the geopotential coefficients provided with ESPOD, they are easily converted to that reference and provide in any case an estimate of the relative influence of various geopotential terms.

The test was performed by taking initial conditions for a 100 nautical mile altitude circular orbit over a (hypothetical) spherical earth and calculating for that orbit the partial derivatives of the cartesian coordinates with respect to the coefficients of the geopotential harmonics. The partial derivatives at the points of 1/4 orbit, 1/2 orbit, 3/4 orbit and one full orbit were root-sum squared to obtain the total effect of the coefficient on each position and velocity. These parial derivatives were then multiplied by the values of the coefficients and uncertainties tabulated in Table 5. These values were derived from W.M. Kaula's report, "NASA Technical Note," D-1848, June 1963.

Two cases were run with the defined orbit, one starting at the north pole on a polar orbit and one starting at the equator on a polar orbit. Tables 6 and 7 show the results of this study.

Table 5. Geopotential Coefficients from NASA Technical Note D-1848, June 1963

n	m	$J_{n,m} \times 10^6$	$\lambda_{n,m}$
2		1082.30 ±0.01	
3		-2.59 ±0.03	
4		-1.65 ±0.30	
5		-0.10 ±0.03	
6		0.36 ±0.07	
7		-0.39 ±0.04	
2	2	1.62 ±0.15	-21°.4 ±2°.8
3	3	0.11 ±0.06	37°.6 ±11°.0
4	4	0.013 ±0.006	28°.4 ±6.5°
3	1	1.92 ±0.23	-3°.6 ±6°.5
3	2	0.12 ±0.09	±28°.4
4	1	0.48 ±0.14	±17°.9
4	2	0.072 +0.042	47°.7 ±16°.9
4	3	0.031 ±0.012	5°.9 ±7°.0

Table 6. Effect of Gravitational Harmonics and Their Uncertainties on a Polar Orbit Starting at North Pole

	At One Fourth Revolution		At One Half Revolution	
	Position, feet	Velocity, ft/sec	Position, feet	Velocity ft/sec
J_2	147, 330 ±1.4	164.71 ±0.002	272, 900 ±2.5	322.42 ±0.003
J_3	261.9 ±3.0	0.30 ±0.003	636.9 ±7.4	0.65 ±0.008
J_4	25.8 ±4.7	0.01 ±0.002	416.2 ±75.7	0.49 ±0.09
J_5	4.6 ±1.4	0.002 ±0.0005	24.3 ±7.3	0.02 ±0.007
J_6	17.5 ±3.4	0.01 ±0.002	87.8 ±17.1	0.10 ±0.02
J_7	22.1 ±2.3	0.03 ±0.003	91.8 ±9.4	0.09 ±0.009
$J_2^{(2)}$	882.8 ±81.7	1.12 ±0.10	455.5 ±42.2	0.52 ±0.05
$J_3^{(1)}$	694.8 ±83.2	0.74 ±0.09	389.0 ±46.6	0.46 ±0.06
$J_3^{(2)}$	37.5 ±28.1	0.01 ±0.01	52.6 ±39.5	0.04 ±0.03
$J_3^{(3)}$	66.8 ±36.4	0.08 ±0.04	184.2 ±100.5	0.20 ±0.11
$J_4^{(1)}$	83.4 ±24.3	0.03 ±0.008	52.3 ±15.3	0.09 ±0.03
$J_4^{(2)}$	66.2 ±38.6	0.09 ±0.05	32.3 ±18.9	0.03 ±0.02
$J_4^{(3)}$	74.9 ±29.0	0.12 ±0.05	89.9 ±34.8	0.08 ±0.03
$J_4^{(4)}$	60.9 ±28.1	0.12 ±0.06	192.3 ±88.8	0.22 ±0.10

	At Three-Fourths Revolution		At One Revolution	
	Position, feet	Velocity, ft/sec	Position, feet	Velocity, ft/sec
J_2	481, 030 ±4.4	515.94 ±0.005	520, 790 ±4.8	616.56 ±0.006
J_3	867.6 ±10.0	0.92 ±0.01	731.5 ±8.5	1.01 ±0.01
J_4	728.3 ±132.4	0.78 ±0.14	797.1 ±144.9	0.94 ±0.17
J_5	31.2 ±9.4	0.03 ±0.01	24.7 ±7.4	0.03 ±0.01
J_6	153.3 ±29.8	0.16 ±0.03	168.1 ±32.7	0.20 ±0.04
J_7	115.3 ±11.8	0.12 ±0.01	87.7 ±9.0	0.13 ±0.01
$J_2^{(2)}$	817.2 ±75.7	0.95 ±0.09	1055.8 ±97.8	1.22 ±0.11
$J_3^{(1)}$	388.6 ±46.6	0.32 ±0.04	90.9 ±10.9	0.11 ±0.01
$J_3^{(2)}$	36.9 ±27.7	0.06 ±0.04	85.3 ±63.9	0.05 ±0.04
$J_3^{(3)}$	271.4 ±148.0	0.24 ±0.13	141.2 ±77.0	0.15 ±0.08
$J_4^{(1)}$	106.6 ±31.1	0.03 ±0.01	11.5 ±3.4	0.15 ±0.04
$J_4^{(2)}$	68.9 ±40.2	0.09 ±0.05	115.2 ±67.2	0.13 ±0.07
$J_4^{(3)}$	85.4 ±33.1	0.07 ±0.03	17.9 ±6.9	0.13 ±0.05
$J_4^{(4)}$	354.9 ±163.8	0.41 ±0.19	474.3 ±218.9	0.54 ±0.25

Table 7. Effect of Gravitational Harmonics and Their Uncertainties on a Polar Orbit Starting at Equator

	At One Fourth Revolution		At One Half Revolution	
	Position, feet	Velocity, ft/sec	Position, feet	Velocity, ft/sec
J_2	22,380 ±0.2	22.07 ±0.0002	56,141 ±0.5	66.98 ±0.0006
J_3	21.7 ±0.3	0.05 ±0.0006	106.0 ±1.2	0.10 ±0.001
J_4	29.9 ±5.4	0.09 ±0.02	229.3 ±41.7	0.27 ±0.05
J_5	1.4 ±0.4	0.004 ±0.001	8.7 ±2.6	0.01 ±0.003
J_6	4.2 ±0.8	0.005 ±0.001	4.0 ±0.8	0.005 ±0.0009
J_7	3.1 ±0.3	0.007 ±0.0008	20.7 ±2.1	0.02 ±0.003
$J_2^{(2)}$	291.4 ±27.0	0.47 ±0.04	1,173.0 ±108.6	1.44 ±0.13
$J_3^{(1)}$	94.0 ±11.3	0.07 ±0.009	502.4 ±60.2	0.60 ±0.07
$J_3^{(2)}$	35.1 ±26.4	0.06 ±0.04	111.2 ±83.4	0.13 ±0.10
$J_3^{(3)}$	88.2 ±48.1	0.11 ±0.06	228.7 ±124.8	0.20 ±0.11
$J_4^{(1)}$	7.9 ±2.3	0.04 ±0.01	71.3 ±20.8	0.04 ±0.01
$J_4^{(2)}$	12.0 ±7.0	0.04 ±0.02	18.1 ±10.6	0.04 ±0.02
$J_4^{(3)}$	67.0 ±25.9	0.10 ±0.04	142.2 ±55.0	0.09 ±0.04
$J_4^{(4)}$	85.6 ±39.5	0.11 ±0.05	262.6 ±121.2	0.35 ±0.16

	At Three-Fourths Revolution		At One Revolution	
	Position, feet	Velocity, ft/sec	Position, feet	Velocity, ft/sec
J_2	101,350 ±0.9	105.41 ±0.001	102,630 ±0.9	122.67 ±0.001
J_3	225.5 ±2.6	0.17 ±0.002	121.6 ±1.4	0.15 ±0.002
J_4	405.0 ±73.6	0.17 ±0.03	421.9 ±76.7	0.50 ±0.09
J_5	12.4 ±3.7	0.002 ±0.0007	5.6 ±1.7	0.007 ±0.002
J_6	8.2 ±1.6	0.009 ±0.002	6.1 ±1.2	0.007 ±0.001
J_7	42.4 ±4.3	0.03 ±0.003	22.5 ±2.3	0.03 ±0.003
$J_2^{(2)}$	2,135.2 ±197.7	2.32 ±0.21	2,431.6 ±225.2	2.90 ±0.27
$J_3^{(1)}$	1065.5 ±127.6	1.22 ±0.15	1248.8 ±149.6	1.25 ±0.15
$J_3^{(2)}$	124.5 ±93.3	0.11 ±0.08	43.2 ±32.4	0.06 ±0.04
$J_3^{(3)}$	192.7 ±105.1	0.18 ±0.10	34.6 ±18.8	0.16 ±0.09
$J_4^{(1)}$	10.3 ±3.0	0.12 ±0.04	132.5 ±38.6	0.001 ±0.0004
$J_4^{(2)}$	56.6 ±33.0	0.06 ±0.04	87.9 ±51.3	0.11 ±0.07
$J_4^{(3)}$	70.6 ±27.3	0.12 ±0.05	113.1 ±43.8	0.03 ±0.01
$J_4^{(4)}$	554.2 ±255.8	0.61 ±0.28	700.9 ±323.5	0.82 ±0.38

4. APPLICATION AND THEORY OF THE A PRIORI A^TA MATRIX (S-MATRIX)

4.1 ESPOD OUTPUT

ESPOD will punch out the A^TA matrix evaluated at epoch following a differential correction by selecting this option with JDC card column 44 set to 1. ESPOD will punch out the A^TA matrix evaluated at an arbitrary time following a differential correction by selecting this option with JDC card columns 51, 55 and 56 set to 1, assuming a correct specification of the arbitrary time.

4.2 GENERAL APPLICATION OF A PRIORI A^TA MATRIX

The A^TA matrix summarizes, scales, and weights the contribution of a set of residuals to the differential correction for a set of orbital elements after an integration through a set of data over a given time span. The A^TA matrix is specific to the time at which the orbital elements are defined. If it is desired to use the same residuals to affect a subsequent correction to the elements, in combination with a newly observed set of residuals, the punched out A^TA matrix permits this application. The time at which the elements to be corrected will be defined must be known and the A^TA matrix for the earlier residuals developed for that time. The A^TA matrix is then provided to ESPOD with preliminary data and by setting JDC card column 42 equal to 1. See Reference 2.5.2, Section 5-12* for further details and restrictions on the use of the a priori A^TA matrix.

4.3 THEORY (WITH EXAMPLE)

4.3.1 Run 1

Let elements E_1 define residuals R_1 from observations O_1 and also define $A_1^TA_1 = S_1$. Together they define a correction $\overline{\Delta E}_1$. Let elements $E_2 = E_1 + \overline{\Delta E}_1$, that is, those elements corrected from E_1 by residuals R_1 on Run 1. Let $A_1^TA_1$ be punched out.

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*See Reference 2.5.2, Section 6.1.4, for the definition of A^TA.

4.3.2 Run 2

Let elements E_2 define residuals $\overline{R}_2$ from observations O_2 and also define $A_2^T A_2$. Use $A_1^T A_1$ as an a priori matrix, which during Run 2 accumulates in the sum $A_1^T A_1 + A_2^T A_2 = \overline{S}_2$. Elements E_2, together with $\overline{R}_2$ and $\overline{S}_2$, define a correction $\overline{\Delta E}_2$. Let elements $\overline{E}_3 = E_2 + \overline{\Delta E}_2$.

4.3.3 Check Run 3

Let elements E_1 define residuals R_1 and R_2 from observations O_1 and O_2 and also define $A_3^T A_3$. Together they define ΔE_1. Let elements $E_3 = E_1 + \Delta E_1$

$$E_3 \cong \overline{E}_3$$

4.3.4 Proof

See Reference 2.5.2, Section 6, for general development of this notation.

$f_1(E)$ = Weighted residual vector, data 1

A_1 = Partials, data 1

$f_2(E)$ = Weighted residual vector, data 2

A_2 = Partials, data 2

Run 1

$$A_1^T A_1 \left(E_2 - E_1\right) = A_1^T f_1 \left(E_1\right)$$

Run 2

$$\left(A_1^T A_1 + A_2^T A_2\right)\left(\overline{E}_3 - E_2\right) = A_2^T f_2 \left(E_2\right) \cong A_2^T f_2 \left(E_1\right) - A_2^T A_2 \left(E_2 - E_1\right)$$

$$\left(A_1^T A_1 + A_2^T A_2\right)\left[\overline{E}_3 - E_1 - \left(E_2 - E_1\right)\right] \cong A_2^T f_2 \left(E_1\right) - A_2^T A_2 \left(E_2 - E_1\right)$$

$$\left(A_1^T A_1 + A_2^T A_2\right)\left(\overline{E}_3 - E_1\right) - A_1^T A_1 \left(E_2 - E_1\right) - A_2^T A_2 \left(E_2 - E_1\right)$$

$$\cong A_2^T f_2 \left(E_1\right) - A_2^T A_2 \left(E_2 - E_1\right)$$

$$\left(A_1^TA_1 + A_2^TA_2\right)\left(\overline{E}_3 - E_1\right) \cong A_2^Tf_2\left(E_2\right) + A_1^TA_1\left(E_2 - E_1\right)$$

$$\left(A_1^TA_1 + A_2^TA_2\right)\left(\overline{E}_3 - E_1\right) \cong A_2^Tf_2\left(E_2\right) + A_1^Tf_1\left(E_1\right)$$

<u>Run 3</u>

$$\left(A_1^TA_1 + A_2^TA_2\right)\left(E_3 - E_1\right) = A_1^Tf_1\left(E_1\right) + A_2^Tf_2\left(E_2\right)$$

Thus, $\overline{E}_3 \cong E_3$

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Third Quarterly Report of a general orbit determination program prepared for use by the SPACETRACK/SPADATS Center, Ent Air Force Base, Colorado Springs, Colorado. Its primary purpose is to determine satellite orbits. It determines the elements of a satellite orbit and a covariance matrix of uncertainty in the determination, starting from some initial estimate of these elements and correcting it in accordance with observational data. It predicts the future position and velocity of the satellite from the best elements obtained.

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